IS COSMIC COINCIDENCE A CONSEQUENCE OF A LAW OF NATURE?

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A field theory is proposed where the regular fermionic matter and the dark fermionic matter are different states of the same "primordial" fermion fields. In regime of the fermion densities typical for normal particle physics, the primordial fermions split into three families identified with regular fermions. When fermion energy density becomes comparable with dark energy density, the theory allows new type of states. The possibility of such Cosmo-Low Energy Physics (CLEP) states is demonstrated by means of solutions of the field theory equations describing FRW universe filled by homogeneous scalar field and uniformly distributed nonrelativistic neutrinos. Neutrinos in CLEP state are drawn into cosmological expansion by means of dynamically changing their own parameters. One of the features of the fermions in CLEP state is that in the late time universe their masses increase as $a^{3/2}$ ($a = a(t)$ is the scale factor). The energy density of the cold dark matter consisting of neutrinos in CLEP state scales as a sort of dark energy; this cold dark matter possesses negative pressure and for the late time universe its equation of state approaches that of the cosmological constant. The total energy density of such universe is less than it would be in the universe free of fermionic matter at all. The (quintessence) scalar field is coupled to dark matter but its coupling to regular fermionic matter appears to be extremely strongly suppressed. The key role in obtaining these results belongs to a fundamental constraint (which is consequence of the action principle) that plays the role of a new law of nature.

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I. INTRODUCTION

Results of cosmological observations [1], [2] strengthen confidence in the truth of a model according to which our universe is spatially flat and composed by 2/3 of the so-called dark energy [3] (responsible for the present accelerated expansion) and by 1/3 of the cold dark matter [4] responsible for the flat rotation curves of galaxies and for structure formation at large scales. The notion of dark energy is usually used as a label for either positive cosmological constant or a scalar field slowly evolving down a potential (quintessence) [5]. The attribute of dark energy is negative pressure. In contrast with this, cold dark matter is expected naturally to be pressureless. This means that dark energy and dark matter energy densities must scale in time in a very different way. If their nearly equal magnitudes in the present day universe is non accidental, this problem, known as "cosmic coincidence problem" [6] appears to be one of the strongest challenge to both cosmology and particle physics.

The wish to combine a possibility to describe an accelerating expansion for the present day universe with a solution of the cosmic coincidence problem in the framework of a single consistent model, was a main motivation of a number of recent attempts to modify the particle physics models underlying the quintessence scenarios. The basic idea of these modifications consists in specific changes of the fundamental (or effective) Lagrangian of the field theory intended to provide possibilities for "over-pumping" of a part of the dark energy into the dark matter energy. Cosmological consequences of a scalar field with exponential potential coupled to matter was discussed in Refs. [7], [8]. One of the popular modifications of the particle physics model is known as variable-mass particles (VAMPs), an idea which was discussed in papers of Refs. [8], [9]. One of the aims of authors of Ref. [9] was to solve the age problem of the Universe. In subsequent papers it was then realized that the desirable effect for the coincidence problem simultaneously with an accelerated expansion can be achieved by assuming that mass of the dark matter particle depends on the (quintessence) scalar field in such a way that the cosmic time evolution of the quintessence field is accompanied by increasing the mass of the dark matter particles. By tuning of the appropriate parameters one can achieve both accelerated expansion and cosmic coincidence. This and other possible realizations of the basic idea during the last three years of the intensive study of this intriguing problem are listed partially in Refs. [10]- [17].

Together with a definitely successful fitting of some of the observational data, the most significant achievement of these models consists in the possibility of providing both the accelerating expansion and a resolution of the co-incidence problem without fine tuning[1]. In spite of this they still have fundamental problem. Although there are some justifications for choices of certain types of dark matter-dark energy coupling in the Lagrangian,[2] there is a necessity to assume [21], [10], [13], [14] the absence or extremely strong suppression of the barion matter-dark energy coupling[3]. Actually this problem was known from the very beginning in the quintessence models since generically there are no reasons for the absence of a direct coupling of the quintessence scalar field ϕ to the barion matter. Such coupling would be the origin of a long range scalar force because of the very small mass of the quintessence field ϕ. This "fifth-force" problem might be solved if there would be a shift symmetry $\phi \rightarrow \phi + const$ of the action [22] which should be a reason for a strong suppression of the direct quintessence - barion matter coupling. However the quintessence potential itself does not possess this symmetry. The situation with the "fifth-force" problem becomes still more critical in the discussed above models since one should explain now why the direct quintessence-dark matter coupling is permissible in the Lagrangian while the same is forbidden for the barion matter. Irrespective of the nature of the dark matter, it is very hard to believe that there may be a symmetry which is responsible for such situation.

The above mentioned modifications of the particle physics models are based on the assumption that all the fields of the fundamental particle theory should be divided into two large groups: one describing detectable particles (ordinary matter) and the other including dark matter particles. The main purpose of this paper is to demonstrate that there is a field theory which is able to propose a resolution for the above problems by an absolutely new way: **the dark matter is not introduced as a special type of matter but rather it appears as the solution of equations of motion describing a new type of states** of the (primordial) neutrino field; in other words, **the dark neutrino matter and the regular neutrino generations (electron, muon and τ neutrinos) are different states of the same primordial fermion field**.

This field theory is the Two Measures Theory (TMT) originally built with the aim to solve the "old" cosmological constant problem [23]- [27] and was not addressed to the above problems from the beginning. The TMT model we continue to study in the present paper possesses spontaneously broken global scale symmetry [27] which includes the

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[1]See however the recent paper [18]

[2]Such justifications are based for example on Brans-Dicke type theories [13] or on strongly coupled string theory [19] or on the large extra dimensions models [20] or on the supersymmetric theories [15]

[3]In terminology of Ref. [13] it is called "choice of dark-dark coupling"

shift symmetry $\phi \rightarrow \phi + const$ and it allows to suggest [28], [29] a simultaneous resolution both of the fermion families problem and of the "fifth-force" problem. **The theory starts from one primordial fermion field for each type of leptons and quarks**, e.g., in $SU(2) \times U(1)$ gauge theory the fermion content is the primordial neutrino and electron fields and primordial u and d quark fields. It turns out that masses and interactions of fermions as well as the structure of their contributions to the energy-momentum tensor depend on the fermion densities.

At a fermion energy density corresponding to normal laboratory particle physics conditions (that we will call "high fermion density"), the fermion energy-momentum tensor is canonical and each of the primordial fermions splits into three different states with different masses (one of these states should be realized via fermion condensate). These states were identified in Refs. [28,29] with the mass eigenstates of the fermion generations, and this effect is treated as the families birth effect. Although the original action includes scale invariant interaction of the dilaton field ϕ (playing the role of the quintessence field) with all primordial fermions, the effective interaction of the dilaton with the regular fermions of the first two generations appears to be extremely suppressed. In other words, the interaction of the dilaton with matter observable in gravitational experiments is practically switched off, and that solves the "fifth-force" problem.

In TMT, physics of fermions at very low densities turns out to be very different from what we know in normal particle physics. The term very low fermion density means here that the fermion energy density is comparable with the dark energy density. In this case, in addition to the canonical contribution to the energy-momentum tensor, each of the primordial fermion fields has a noncanonical contribution in the form of a **dynamical fermionic Λ term**.

If the fermion energy density is comparable with the dark energy density then as will be shown in the present paper, the theory predicts that the primordial fermion may not split into generations and in the FRW universe it can participate in the expansion of the universe by means of changing its own parameters. We call this effect "Cosmo-Particle Phenomenon" and refer to such states as Cosmo-Low Energy Physics (CLEP) states.

As the first step in studying Cosmo-Particle Phenomena, in this paper we restrict ourselves to the consideration of a simplified cosmological model where universe is filled by a homogeneous scalar field ϕ and uniformly distributed non-relativistic (primordial) neutrinos and antineutrinos in CLEP states. Such CLEP-neutrino matter is detectable practically only through gravitational interaction and this is why it can be regarded as a model of dark matter. The mass of CLEP-neutrino increases as $a^{3/2}$ where $a = a(t)$ is the scale factor. This dark matter is also cold one in the sense that kinetic energy of neutrinos is negligible as compared to their mass. However due to the dynamical fermionic Λ term generated by neutrinos in CLEP state, this cold dark matter has negative pressure and its equation of state approaches $p_{d.m.} = -\rho_{d.m.}$ as $a(t) \rightarrow \infty$. Besides, the energy density of this dark matter scales in a way very similar to the dark energy which includes both a cosmological constant and an exponential potential. So, due to the Cosmo-Particle Phenomena, TMT allows the universe to achieve both the accelerated expansion and cosmic coincidence without the need to postulate the existence of a special sort of matter called "dark matter". The remarkable feature of such a Cosmo-Particle solution is that **the total energy density of the universe in this case is less than it would be in the universe free of fermionic matter at all**.

All these unusual effects of TMT are due to the existence of a fundamental constraint that emerges as a consistency condition of the equations of motion. According to the role of this constraint it can be regarded as a new law of nature.

Some notions about the content and organization of the paper are needed. Plenty of new ideas have obliged us to simplify the presentation of the theory as far as possible. For this reason we ignored gauge non-abelian structure as well as Higgs fields and Higgs phenomena, although they were already presented in our earlier publication [29]. We also ignore here different possible ways for introducing neutrino mass term and describe it in the simplest "naive" way because this item has no direct relation to questions studied in this paper. We are not concern also a number of important astrophysical features of the Cosmo-Particle Phenomena, such as clustering properties of neutrinos in CLEP states. As for the presentation of TMT itself (that requires explanation quite a number of ideas and calculations developed in previous publications [23]- [26]), we have tried to make the paper self-sufficient. Main ideas of TMT and some technical details one can find in Sec.II and in Appendixes B and C. Besides, in Sec.II and in Appendix A we present some brane models arguments in favor of the general structure of the action in TMT.

II. MAIN IDEAS OF THE TWO MEASURES THEORY

The Two Measures Theory (TMT) is a generally coordinate invariant theory where the action has the form

$$S = \int L_1 \Phi d^4 x + \int L_2 \sqrt{-g} d^4 x \tag{1}$$

including two Lagrangians L_1 and L_2 and two measures of the volume elements (Φd^4x and $\sqrt{-g}d^4x$ respectively). One is the usual measure of integration $\sqrt{-g}$ in the 4-dimensional space-time manifold equipped by the metric $g_{\mu\nu}$. Another is also a scalar density built of four scalar fields φ_a ($a = 1, 2, 3, 4$)

$$\Phi = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\mu\varphi_a\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d. \tag{2}$$

Since Φ is a total derivative, a shift of L_1 by a constant, $L_1 \rightarrow L_1 + const$, has no effect on the equations of motion. Similar shift of L_2 would lead to the change of the constant part of the Lagrangian coupled to the volume element $\sqrt{-g}d^4x$. In the standard General Relativity (GR), this constant term is the cosmological constant. However in TMT the relation between the constant term of L_2 and the physical cosmological constant is very non trivial (see [26], [27], [30], [31]).

It is assumed that the Lagrangians L_1 and L_2 are functions of the matter fields, the dilaton field, the metric, the connection (or spin-connection) but not of the "measure fields" φ_a. In such a case, i.e. when the measure fields φ_a enter in the theory only via the measure Φ, the action (1) has the infinite dimensional symmetry [26]: $\varphi_a \rightarrow \varphi_a + f_a(L_1)$, where $f_a(L_1)$ is an arbitrary function of L_1.

Varying the measure fields φ_a, we get

$$B_a^\mu\partial_\mu L_1 = 0 \tag{3}$$

where

$$B_a^\mu = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d. \tag{4}$$

Since $Det(B_a^\mu) = \frac{4^{-4}}{4!}\Phi^3$ it follows that if $\Phi \neq 0$,

$$L_1 = sM^4 = const \tag{5}$$

where $s = \pm 1$ and M is a constant of integration with the dimension of mass.

Notice the **very important difference of TMT from scalar-tensor theories with nonminimal coupling**: if the field Φ were the fundamental (non composite) one then the variation of Φ would results in the equation $L_1 = 0$.

Important feature of TMT that is responsible for many interesting and desirable results of the field theory models studied so far [23]- [31] consists of the assumption that all fields, including also metric, connection (or vierbein and spin-connection) and the **measure fields** φ_a are independent dynamical variables. All the relations between them are results of equations of motion. Such understanding of the variational principle may be regarded as a generalization of the usual variational principle to the case when theory contains two measures of integration and two Lagrangians. In particular, the independence of the metric and the connection means that we proceed in the first order formalism and the relation between connection and metric is not necessarily according to Riemannian geometry. Applying the Palatini formalism in TMT one can show (see for example [26] or Appendix C of the present paper) that the resulting relation between metric and connection includes also the gradient of the ratio of the two measures

$$\zeta \equiv \frac{\Phi}{\sqrt{-g}} \tag{6}$$

which is a scalar field. By an appropriate change of the dynamical variables which includes a conformal transformation of the metric, one can formulate the theory as that in a Riemannian (or Riemann-Cartan) space-time. The corresponding conformal frame we call "the Einstein frame". The big advantage of TMT is that in the very wide class of models, *the equations of motion take the canonical form of those of GR*, including the field theory models in curved space-time. One has to stress that **TMT has nothing common with any sort of the familiar scalar-tensor theories**. All novelty of TMT in the Einstein frame as compared with the standard GR is revealed only in an unusual structure of the effective potentials and interactions. The key role in this effect belongs to the scalar field $\zeta(x)$. Namely, the consistency condition of equations of motion has the form of algebraic constraint which determines $\zeta(x)$ as a function of matter fields. The surprising feature of the theory is that neither Newton constant nor curvature appears in this constraint which means that the *geometrical scalar field* $\zeta(x)$ is determined by the matter fields configuration locally and straightforward (that is without gravitational interaction). As a result of this, $\zeta(x)$ has a decisive influence in the determination of the effective (that is appearing in the Einstein frame) interactions and particle masses, and due to this, in the particle physics, cosmology and astrophysics.

As for the possible origin of the modified measure one can think of the following arguments. Let us start by noticing that the modified measure part of the action (1), $\int L_1\Phi d^4x$, in the case $L_1 = const$ becomes a topological contribution to the action, since Φ is a total derivative. It is very interesting that this structure can be obtained by considering the topological theory which results from studying "space-time filling branes", Refs. [32]- [34].

Following for example Ref. [33], the Nambu-Goto action of a 3-brane embedded in a 4-dimensional space-time[4]

$$S_{NG} = T \int d^4x \sqrt{|\det(\partial_\mu \varphi^a \partial_\nu \varphi^b G_{ab}(\varphi))|} \qquad (7)$$

(where $G_{ab}(\varphi)$ is the metric of the embedding space), is reduced to

$$\frac{T}{4!} \int d^4x \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \sqrt{|\det G_{ab}(\varphi^a)|} = \frac{T}{4!} \int \Phi \sqrt{|\det G_{ab}(\varphi^a)|} d^4x. \qquad (8)$$

where Φ is determined by Eq.(2). The Euler-Lagrange equations for φ^a are identities which is the result of the fact [33] that the action (8) is topological.

This is the action of a "pure" space-filling brane governed just by the identically constant brane tension. Let us now assume that the 3-brane is equipped with its own metric $g_{\mu\nu}$ and connection. If we want to describe a space-filling brane where gravity and matter are included by means of the Lagrangian $L_1 = L_1(g_{\mu\nu}, connection, matter fields)$, we are led to the following form of the brane contribution to the action

$$S_1 = \int L_1 \Phi \sqrt{|\det G_{ab}(\varphi^a)|} d^4x \qquad (9)$$

Recall that $\sqrt{|\det G_{ab}(\varphi^a)|}$ is a brane scalar.

If L_1 is not identically a constant (we have seen above that it may become a constant on the "mass shell", i.e. when equations of motion are satisfied), then we are not talking any more of a topological contribution to the action. Assuming again that L_1 is φ^a independent, it is easy to check that in spite of emergence of an additional factor $\sqrt{|\det G_{ab}(\varphi^a)|}$ in Eq.(9) as compared with the first term in Eq.(1), variation of φ^a yields exactly the same equation (5) if $\Phi \neq 0$. Moreover, all other equations of motion of the two measures theory remains unchanged. The only effect of this additional factor consists in the redefinition of the scalar field ζ, Eq.(6), where $\sqrt{|\det G_{ab}(\varphi^a)|}$ emerges as an additional factor. Notice that the same results are obtained if instead of $\sqrt{|\det G_{ab}(\varphi^a)|}$ in Eq.(9) there will be arbitrary function of φ^a.

More arguments in favor of the general structure of the action in TMT are given in Appendix A.

III. SIMPLIFIED SCALE-INVARIANT MODEL

The TMT models possessing a global scale invariance [27–29] are of significant interest because they demonstrate the possibility of spontaneous breakdown of the scale symmetry[5]. This effect appears due to the fact that the measure Φ is a total derivative. One of the interesting applications of the scale invariant TMT models [28] is a possibility to generate the exponential potential for the scalar field ϕ by means of the mentioned spontaneous symmetry breaking without introducing any potentials of ϕ in the Lagrangians L_1 and L_2 in the action (1). Some cosmological applications of this effect have been also studied in Ref. [28]. Another application [28,29], rather surprising one, is the possibility to suggest the way for the resolution of the fermion families problem and at the same time to solve the fifth force problem.

The matter content of the simplified model we study here includes the scalar field ϕ, two so-called primordial fermion fields (the neutrino primordial field ν and the electron primordial field E) and electromagnetic field A_μ. The latter is included in order to demonstrate that gauge fields have no direct relation to the effects studied in this paper. Generalization to the non-Abelian gauge models including Higgs fields and quarks is straightforward [29]. To simplify

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[4]It should be pointed out that string and brane theories can be formulated with the use of a modified measure [35] giving some new interesting results. In the spirit of the interpretation developed in this section, such strings or branes could be regarded as extended objects moving in an embedding extended object of the same dimensionality.

[5]The field theory models with explicitly broken scale symmetry and their application to the quintessential inflation type cosmological scenarios have been studied in Ref. [30]. Inflation and transition to slowly accelerated phase from higher curvature terms was studied in Ref. [31].

the presentation of the features of TMT studied in this paper we ignore also the chiral properties of neutrino; this can be done straightforward and does not affect the results obtained in this paper.

Keeping the general structure (1), it is convenient to represent the action in the following form:

$$S = \int d^4x e^{\alpha\phi/M_p}(\Phi + b\sqrt{-g})\left[-\frac{1}{\kappa}R(\omega,e) + \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right] - \int d^4x e^{2\alpha\phi/M_p}[\Phi V_1 + \sqrt{-g}V_2]$$

$$+ \int d^4x e^{\alpha\phi/M_p}(\Phi + k\sqrt{-g})\frac{i}{2}\left[\overline{\nu}\left(\gamma^a e_a^\mu \overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu \gamma^a e_a^\mu\right)\nu + \overline{E}\left(\gamma^a e_a^\mu \overrightarrow{\nabla}_\mu^{(E)} - \overleftarrow{\nabla}_\mu^{(E)}\gamma^a e_a^\mu\right)E\right]$$

$$- \int d^4x e^{\frac{3}{2}\alpha\phi/M_p}\left[(\Phi + h_\nu\sqrt{-g})\mu_\nu\overline{\nu}\nu + (\Phi + h_E\sqrt{-g})\mu_E\overline{E}E\right] - \int d^4x\sqrt{-g}\frac{1}{4}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu} \quad (10)$$

where V_1 and V_2 are constants, $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, μ_ν and μ_E are the mass parameters,
$\overrightarrow{\nabla}_\mu = \overrightarrow{\partial}_\mu + \frac{1}{2}\omega_\mu^{cd}\sigma_{cd}$, $\overleftarrow{\nabla}_\mu = \overleftarrow{\partial}_\mu - \frac{1}{2}\omega_\mu^{cd}\sigma_{cd}$, $\overrightarrow{\nabla}_\mu^{(E)} = \overrightarrow{\partial}_\mu + \frac{1}{2}\omega_\mu^{cd}\sigma_{cd} + ieA_\mu$ and
$\overleftarrow{\nabla}_\mu^{(E)} = \overleftarrow{\partial}_\mu - \frac{1}{2}\omega_\mu^{cd}\sigma_{cd} - ieA_\mu$; $R(\omega,e) = e^{a\mu}e^{b\nu}R_{\mu\nu ab}(\omega)$ is the scalar curvature,
e_a^μ and ω_μ^{ab} are the vierbein and spin-connection respectively; $g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab}$ and

$$R_{\mu\nu ab}(\omega) = \partial_\mu\omega_{\nu ab} + \omega_{\mu a}^c\omega_{\nu cb} - (\mu \leftrightarrow \nu). \quad (11)$$

The action (10) is invariant under the global scale transformations

$$e_\mu^a \to e^{\theta/2}e_\mu^a, \quad \omega_{ab}^\mu \to \omega_{ab}^\mu, \quad \varphi_a \to \lambda_a\varphi_a \quad where \quad \Pi\lambda_a = e^{2\theta}$$

$$A_\alpha \to A_\alpha, \quad \phi \to \phi - \frac{M_p}{\alpha}\theta, \quad \Psi_i \to e^{-\theta/4}\Psi_i, \quad \overline{\Psi}_i \to e^{-\theta/4}\overline{\Psi}_i \quad (12)$$

where Ψ_i $(i = \nu, E)$ is the general notation for the primordial fermion fields ν and E.

In (10) there are two types of the gravitational terms and of the "kinetic-like terms" (both for the scalar field and for the primordial fermionic ones) which respect the scale invariance : the terms of the one type coupled to the measure Φ and those of the other type coupled to the measure $\sqrt{-g}$. One should point out also the possibility of introducing two different potential-like exponential functions coupled to the measures Φ and $\sqrt{-g}$ with constants V_1 and V_2 respectively[6] . For the same reason there are two different sets of the mass-like terms of the primordial fermions. Constants b, k, h_ν, h_E are non specified dimensionless real parameters of the model and we will only assume that *all these parameters are of the same order of magnitude*. The real positive parameter α is assumed to be of the order of one.

The choice of the action (10) needs a few additional explanations:

1) In order to avoid a possibility of negative energy contribution from the space-time derivatives of the dilaton ϕ (see Ref. [28]) we have chosen the coefficient b in front of $\sqrt{-g}$ in the first integral of (10) to be a common factor of the gravitational term $-\frac{1}{\kappa}R(\omega,e)$ and of the kinetic term for ϕ. This guarantees that this item can not be an origin of ghosts in quantum theory.

2) For the same reasons we choose the kinetic term of A_μ in the conformal invariant form which is possible if this term is coupled only to the measure $\sqrt{-g}$. Introducing the coupling of this term to the measure Φ would lead to a nonlinear equation and non positivity of the energy.

3) One can show that in more realistic theories, like electro-weak one, the right chiral structure in the Einstein frame demands the coupling of the kinetic terms of all the left and right primordial fermions to the measures to be universal. In our simplified version of the theory, this feature is displayed in the choice of the parameter k to be the common factor in front of the corresponding kinetic terms of the both primordial fermions.

Except for these three items, Eq.(10) describes **the most general TMT action satisfying the formulated above symmetries**.

IV. CONSTRAINT AND EQUATIONS OF MOTION IN THE EINSTEIN FRAME

Variation of the measure fields φ_a with the condition $\Phi \neq 0$ leads, as we have already seen in Sec.II, to the equation $L_1 = sM^4$ where L_1 is now defined, according to Eq. (1), as the part of the integrand of the action (10) coupled to

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[6]In more realistic models instead of these constants there will be functions of the Higgs field (see Ref. [29])

the measure Φ. It can be noticed that the appearance of a nonzero integration constant sM^4 spontaneously breaks the scale invariance (12). The explicit form of the equation $L_1 = sM^4$ is presented in Appendix B, Eq.(B1) as well as all other equations of motion. In Appendix C one can find explicit solution for the spin-connection. One can see that the measure Φ degrees of freedom appear in all the equations of motion only through dependence on the scalar field ζ. In particular, the gravitational and all matter fields equations of motion include noncanonical terms proportional to $\partial_\mu \zeta$.

It turns out that with the set of the new variables (ϕ and A_μ remain the same)

$$\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_P}(\zeta + b)g_{\mu\nu}, \quad \tilde{e}_{a\mu} = e^{\frac{1}{2}\alpha\phi/M_P}(\zeta + b)^{1/2}e_{a\mu}, \quad \nu' = e^{-\frac{1}{4}\alpha\phi/M_P}\frac{(\zeta + k)^{1/2}}{(\zeta + b)^{3/4}}\nu, \quad E' = e^{-\frac{1}{4}\alpha\phi/M_P}\frac{(\zeta + k)^{1/2}}{(\zeta + b)^{3/4}}E$$
(13)

which we call the Einstein frame, the spin-connections (C4) become those of the Einstein-Cartan space-time and the noncanonical terms proportional to $\partial_\mu \zeta$ disappear from all equations of motion. Since $\tilde{e}_{a\mu}$, ν' and E' are invariant under the scale transformations (12), spontaneous breaking of the scale symmetry (12) (by means of Eq.(5)) is reduced in the new variables to the **spontaneous breaking of the shift symmetry** $\phi \to \phi + const$.

The gravitational equations (B11) after the change of variables (13) to the Einstein frame and some simple algebra, take the standard GR form

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{\kappa}{2}T_{\mu\nu}^{eff}$$
(14)

$$T_{\mu\nu}^{eff} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + \tilde{g}_{\mu\nu}V_{eff}(\phi;\zeta) + T_{\mu\nu}^{(em)} + T_{\mu\nu}^{(ferm,can)} + T_{\mu\nu}^{(ferm,noncan)}$$
(15)

Here $G_{\mu\nu}(\tilde{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian space-time with the metric $\tilde{g}_{\mu\nu}$; the function $V_{eff}(\phi;\zeta)$ has the form

$$V_{eff}(\phi;\zeta) = \frac{b\left(sM^4e^{-2\alpha\phi/M_P} + V_1\right) - V_2}{(\zeta + b)^2};$$
(16)

$T_{\mu\nu}^{(em)}$ is the canonical energy momentum tensor for the electromagnetic field; $T_{\mu\nu}^{(ferm,can)}$ is the canonical energy momentum tensor for (primordial) fermions ν' and E' in curved space-time including also standard electromagnetic interaction of E'. $T_{\mu\nu}^{(ferm,noncan)}$ is the *noncanonical* contribution of the fermions into the energy momentum tensor

$$T_{\mu\nu}^{(ferm,noncan)} = -\tilde{g}_{\mu\nu}[F_\nu(\zeta)\overline{\nu'}\nu' + F_E(\zeta)\overline{E'}E'] \quad \equiv \quad \tilde{g}_{\mu\nu}\Lambda_{dyn}^{(ferm)}$$
(17)

where

$$F_i(\zeta) \equiv \frac{\mu_i}{2(\zeta + k)^2(\zeta + b)^{1/2}}[\zeta^2 + (3h_i - k)\zeta + 2b(h_i - k) + kh_i], \qquad i = \nu', E'.$$
(18)

The structure of $T_{\mu\nu}^{(ferm,noncan)}$ shows that it behaves as a sort of variable cosmological constant [36] but in our case it is originated by fermions. This is why we will refer to it as **dynamical fermionic Λ term**. This fact is displayed explicitly in Eq.(17) by defining $\Lambda_{dyn}^{(ferm)}$. One has to emphasize the substantial difference of the way $\Lambda_{dyn}^{(ferm)}$ emerges here as compared to the models of the condensate cosmology (see for example Refs. [37]- [39]). In those models the dynamical cosmological constant resulted from bosonic or fermionic condensation. In TMT model studied here, $\Lambda_{dyn}^{(ferm)}$ is originated by fermions but there is no need in any condensate. As we will see in the next sections, Λ_{dyn}^{ferm} becomes negligible in gravitational experiments with observable matter. However it may be very important for some astrophysics and cosmology problems and in particular it plays a key role in the resolution of the cosmic coincidence and other problems discussed in the Introduction.

The dilaton ϕ field equation (B12) in the new variables reads

$$\Box\phi - \frac{\alpha}{M_p(\zeta + b)}\left[sM^4e^{-2\alpha\phi/M_p} - \frac{(\zeta - b)V_1 + 2V_2}{\zeta + b}\right] = -\frac{\alpha}{M_p}[F_\nu\overline{\nu'}\nu' + F_E\overline{E'}E'],$$
(19)

where $\Box\phi = (-\tilde{g})^{-1/2}\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi)$.

Equations (B6),(B7) for the primordial leptons in terms of the variables (13) take the standard form of fermionic equations for ν', E' in the Einstein-Cartan space-time where the standard electromagnetic interaction presents also. All the novelty consists of the form of the ζ depending "masses" $m_i(\zeta)$ of the primordial fermions ν', E':

$$m_i(\zeta) = \frac{\mu_i(\zeta + h_i)}{(\zeta + k)(\zeta + b)^{1/2}} \qquad i = \nu', E'. \tag{20}$$

It should be noticed that change of variables we have performed by means of Eq.(13) provide also a conventional form of the covariant conservation law of fermionic current $j^\mu = \overline{\Psi}' \gamma^a \tilde{e}^\mu_a \Psi'$.

The scalar field ζ in the above equations is defined by the constraint determined by means of Eqs. (B9),(B10) which in the new variables (13) takes the form

$$-\frac{1}{(\zeta + b)^2} \left\{ (\zeta - b) \left[sM^4 e^{-2\alpha\phi/M_p} + V_1 \right] + 2V_2 \right\} = F_\nu(\zeta)\overline{\nu'}\nu' + F_E(\zeta)\overline{E'}E' \quad \equiv \quad -\Lambda^{(ferm)}_{dyn} \tag{21}$$

Notice that neither electromagnetic field nor kinetic terms enter into the constraint. One should point out the interesting and very important fact: the same functions $F_i(\zeta)$ $(i = \nu', E')$ emerge in $\Lambda^{(ferm)}_{dyn}$, in the effective coupling of the dilaton to fermions and in the constraint.

Applying constraint (21) to Eq.(19) one can reduce the latter to the form

$$\Box\phi - \frac{2\alpha\zeta}{(\zeta + b)^2 M_p} sM^4 e^{-2\alpha\phi/M_p} = 0, \tag{22}$$

where ζ is a solution of the constraint (21).

Generically the constraint (21) determines ζ as a very complicated function of ϕ, $\overline{\nu'}\nu'$ and $\overline{E'}E'$. Substituting the appropriate solution for ζ into the above equations of motion one can conclude that in general, there is no sense, for example, to regard $V_{eff}(\phi; \zeta)$, Eq.(16), as the effective potential for the scalar field ϕ because it depends in a very nontrivial way on $\overline{\nu'}\nu'$ and $\overline{E'}E'$ as well. For the same reason, the dynamical fermionic $\Lambda^{(ferm)}_{dyn}$ term, as well as the effective fermion "mass", Eq.(20), describe, in general, self-interactions of the primordial fermions depending also on the scalar field ϕ. Therefore *it is impossible, in general, to separate the terms of $T_{\mu\nu}$ which describe the scalar field ϕ effective potential from the fermion contributions.* Such mixing of the scalar field ϕ and fermionic matter gives rise to a rather complicated system of equations when trying to apply the theory to a general situations that can appear in particle physics, astrophysics and cosmology.

One can note, however, that if for some reasons ζ occurs to be constant of the order of the parameter b then the l.h.s. of the constraint (21) is of the same order of magnitude as $V_{eff}(\phi; \zeta)$, Eq.(16), which in this case is the effective potential of the dilaton field ϕ. We will see that due to our assumption that all the dimensionless parameters in the action (10) are of the same order of magnitude, ζ is in fact of the order of those parameters in the relevant cosmological solutions studied below. So, roughly speaking, **the constraint describes a balance** (in orders of magnitude) which must be realized, according to TMT, **between the dark energy density, on the one hand, and the dynamical fermionic $\Lambda^{(ferm)}_{dyn}$ term, on the other hand**. One has to remind here that the constraint dictating this balance is a consistency condition of equations of motion, from the mathematical point of view. From the physics point of view, the constraint emerges as a new law of nature[7].

There are a few very important particular situations where the theory allows exact solutions of great interest [28], [29]. We will see that for the late time universe, the constraint provides a possibility to explain the cosmic coincidence.

V. DARK ENERGY IN THE ABSENCE OF MASSIVE FERMIONS

In the case of the complete absence of massive fermions the constraint determines ζ as the function of ϕ

$$\zeta = b - \frac{2V_2}{V_1 + sM^4 e^{-2\alpha\phi/M_p}}. \tag{23}$$

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[7]It is interesting to note that on the basis of the observational data, general arguments have been given [40] in favor of the existence of a fundamental principle that correlates somehow different contributions to the energy density of the universe.

The effective potential of the scalar field ϕ results then from Eq.(16)

$$V_{eff}^{(0)}(\phi) \equiv V_{eff}(\phi; \zeta)|_{\overline{\psi'}\psi'=0} = \frac{(V_1 + sM^4 e^{-2\alpha\phi/M_p})^2}{4[b(V_1 + sM^4 e^{-2\alpha\phi/M_p}) - V_2]} \tag{24}$$

and the ϕ-equation (19) is reduced to

$$\Box\phi + V_{eff}^{(0)\prime}(\phi) = 0, \tag{25}$$

where prime sets derivative with respect to ϕ.

Applying this model to the cosmology of the late time universe and assuming that the scalar field $\phi \to \infty$ as $t \to \infty$, we see that the evolution of the late time universe is governed by the sum of the cosmological constant

$$\Lambda^{(0)} = \frac{V_1^2}{4(bV_1 - V_2)} \tag{26}$$

and the quintessence-like scalar field with the potential

$$V_{quint}^{(0)}(\phi) = \frac{(bV_1 - 2V_2)sV_1 M^4 e^{-2\alpha\phi/M_p} + (bV_1 - V_2)M^8 e^{-4\alpha\phi/M_p}}{4(bV_1 - V_2)[b(V_1 + sM^4 e^{-2\alpha\phi/M_p}) - V_2]}. \tag{27}$$

The cosmological constant $\Lambda^{(0)}$ is the asymptotic value (as $t \to \infty$) of the dark energy density

$$\rho_{d.e}^{(0)} = \frac{1}{2}\dot{\phi}^2 + \Lambda^{(0)} + V_{quint}^{(0)}(\phi). \tag{28}$$

for the FRW universe in the model where massive fermions absent. $\Lambda^{(0)}$ is positive provided

$$bV_1 > V_2 \tag{29}$$

that will be assumed in what follows.

There are two ways to provide the observable order of magnitude of the vacuum energy density by an appropriate choice of the parameters of the theory.

(a) The first one consists of the assumption that the parameter b is of the order of unity; then there is no need for V_1 and V_2 to be small: it is enough that $V_2 < 0$ and $V_1 \ll |V_2|$. This possibility is a kind of a **seesaw** mechanism (see [27]). For instance, if $V_1 \sim (10^3 GeV)^4$ and $V_2 \sim (10^{15} GeV)^4$ then $\Lambda^{(0)} \sim (10^{-3} eV)^4$.

(b) The second possibility is to choose the dimensionless parameter $b > 0$ to be very large. In this case the order of magnitude of V_1 and V_2 could be either as in the above case (a) or to be not too much different (or even of the same order).

The mass of a test fermion, that is the primordial fermion field in a state having negligible contributions into equations (15), (19), (21), is ϕ-dependent as it is determined by Eqs.(20) and (23). The asymptotic value of the test fermion mass as $\phi \to \infty$ is very small

$$m_i^{(test)}|_{\zeta=b-2V_2/V_1} = \frac{\mu_i}{\sqrt{2}} \frac{[(b+h_i)V_1 - 2V_2]V_1^{1/2}}{[(b+k)V_1 - 2V_2](bV_1 - V_2)^{1/2}} \qquad i = \nu', E' \tag{30}$$

because it is proportional to $(\Lambda^{(0)}/V_1)^{1/2}$.

VI. REGULAR FERMION (LEPTON) FAMILIES BIRTH EFFECT

We are going here to demonstrate the effect of splitting of each type of the primordial fermions into three generations in the regime where fermion densities are of the typical laboratory particle physics scales. This effect has been discussed in Ref. [29] for lepton and quark primordial fields in the framework of models with massless neutrino. Namely, when the fermion (in particular, neutrino) primordial field is introduced into the theory in such a way that it remains massless then it does not appear in the constraint. As a result, in such model there is no splitting of the primordial

neutrino into three generations. In the present model the primordial neutrino field is regarded on the same manner as the primordial electron field E.

Consider the constraint (21) in a typical particle physics situation, say detection of a single fermion, i.e. a free one particle state of the field Ψ' (which in our simplified model can be either E' or ν'). This measurement implies a localization of the fermion which is expressed in developing a very big value of $|\overline{\Psi'}\Psi'|$ (for the cases when $\zeta = const$, in the local particle rest frame, $\overline{\Psi'}\Psi'$ is, as a matter of fact, the probability density). There are two ways to provide a balance dictated by the constraint:

either

$$F_i(\zeta) \approx 0, \qquad i = \nu', E' \tag{31}$$

or

$$\zeta + b \approx 0. \tag{32}$$

Eq.(31) determines two sets of the constant solutions for ζ

$$\zeta_{1,2}^{(i)} = \frac{1}{2}\left[k - 3h_i \pm \sqrt{(k-3h_i)^2 + 8b(k-h_i) - 4kh_i}\right], \qquad i = \nu', E' \tag{33}$$

They correspond to two different states of the primordial leptons with **different constant masses** determined by Eq.(20) where we have to substitute $\zeta_{1,2}^{(i)}$ instead of ζ. These two states can be identified with the mass eigenstates of *the first two generations of the regular leptons.*

Comparing the simplified scale invariant model we have restricted ourselves in the present paper with the more realistic non Abelian gauge invariant models [29] including Higgs fields, one can see that the parameters V_1 and V_2 in the action (10) have a sense of the Higgs field potentials contributing to the constraint and to the dark energy density. On the basis of the above analysis we conclude that each type of **the primordial fermion fields** (in the present model: ν' and E') **turn into regular fermions of the first two generations, only if its canonical energy density $\tilde{g}^{00}T_{00}^{(ferm,can)}$ is much greater than** $\Lambda_{dyn}^{(ferm)} \propto F_i(\zeta) \approx 0$ **being (as it is dictated by the constraint) of the order of the typical contributions into the dark energy density from the dilaton and Higgs fields**. For example, if the primordial neutrino is in the state with $\zeta = \zeta_1^{(\nu)}$ (or $\zeta = \zeta_2^{(\nu)}$) determined by Eq.(33), it is detected as the regular electron neutrino ν_e (or as the muon neutrino ν_μ) respectively. The states of the primordial electron field E with $\zeta = \zeta_1^{(E)}$ and $\zeta = \zeta_2^{(E)}$ are detected as the regular electron e and muon μ respectively. Similar identification is true also for quarks [29].

The third solution, Eq.(32), we associate with the third fermion generation. It becomes singular if one takes it as an exact equality (this can be seen from equations of motion). This means that one can not neglect the rest of the terms in the constraint (21). Then by solving $\zeta + b$ in terms of ϕ and $\overline{\Psi'_i}\Psi'_i$ ($i = \nu', E'$) we have approximately $\zeta = \zeta_3 \approx -b$ and

$$\frac{1}{\sqrt{\zeta_3 + b}} \approx \left[\frac{\mu_i(b-h_i)\overline{\Psi'_i}\Psi'_i}{4(b-k)\left[b\left(sM^4 e^{-2\alpha\phi/M_p} + V_1\right) - V_2\right]}\right]^{1/3}, \qquad i = \nu', E'. \tag{34}$$

Inserting this into Eq.(20) we see that the fermion self-interaction term in the effective Lagrangian

$$L_{selfint.}^{(3-rd\,family)} \approx \left[\frac{\mu_i^4(b-h_i)^4(\overline{\Psi'_i}\Psi'_i)^4}{4(b-k)^4\left[b\left(sM^4 e^{-2\alpha\phi/M_p} + V_1\right) - V_2\right]}\right]^{1/3}, \qquad i = \nu', E'. \tag{35}$$

is produced in the regime (32). This fermion self-interaction is non-polynomial with dimensionless coupling constant. A full treatment of the third generation requires the study of quantum corrections and fermion condensates [41], [42] which will give the third fermion family appropriate masses. Notice that in contrast with the first two generations, the noncanonical part of the primordial fermion energy density in the regime (32) is non negligible:

$$\Lambda_{dyn}^{ferm}|_{3-rd\,family} = -\frac{1}{2}L_{selfint.}^{(3-rd\,family)} \tag{36}$$

An interesting question in this theory is: how family mixing appears? Or in other words, what is the origin of the Cabbibo angle and more generically of the Kobayashi-Maskawa mass matrix in this theory? We now discuss one possible origin of mixing in TMT.

It is clear that the mass eigenstates defined by Eq.(33) and the one defining the third generation, Eq.(34) (which does not get a mass at the classical level yet) are associated with configurations where a "pure ν" state or a "pure E" state , etc. are considered. Such one-particle states are achieved as free asymptotic states (far from interaction space-time region). In contrast to this, in the interaction regions, different types of primordial fermions are present and therefore the values of ζ in these regions are different from those determined by Eqs.(33) and (34) defining the mass eigenstates. So, it follows that the fermion states participating at the vertices of interaction in general *do not* coincide with the fermion mass eigenstates and from this one can expect the existence of particle mixing, i.e. a Cabbibo angle or more generically, a Kobayashi-Maskawa matrix with nondiagonal elements. This is the question of great interest and it should be studied in the future developments of TMT.

VII. RESOLUTION OF THE FIFTH FORCE PROBLEM

One can show that both in the cosmological scenario without fermions (as in Sec.V) and for the universe filled by fermionic matter (see the next section), the mass of the dilaton (quintessence) field ϕ is extremely small. Then we must explain why there is no a direct coupling of the dilaton to the observed matter (or why it is extremely suppressed), otherwise a long-range scalar force should be observed in gravitational experiments in contradiction with the equivalence principle.

According to the dilaton equation (19), there exists Yukawa-like coupling of the primordial fermions to the dilaton with coupling "constants" $\frac{\alpha}{M_p}F_i(\zeta)$; this coupling becomes usual Yukawa coupling when $\zeta = const$. Recall that to simplify the presentation of the theory we have not included the quark primordial fields. In the complete formulation of the theory the quark primordial fields enter on the same manner as leptonic. Therefore quarks also possess the mentioned Yukawa-like coupling to the dilaton where the functions $F_q(\zeta)$ of the same form as Eq.(18) enter into the same form coupling "constants" we have obtained for the dilaton-lepton coupling.

Similar to what was explained in the previous section, the quark families birth effect occurs at high magnitude of $|\overline{\Psi}'_q\Psi'_q|$ corresponding to the laboratory conditions of the normal particle physics: the balance dictated by the constraint requires then either $F_q(\zeta) \approx 0$ (i.e. ζ is a constant determined by equations of the form of Eq.(33) and corresponding to the first two quark generations) or $\zeta + b \approx 0$ (the third generation).

Practically all fermionic matter observable in gravitational experiments consists of the regular quarks and leptons of the first two generations. In the context of TMT this means that only those primordial fermions participate in gravitational experiments which are in the state with $F(\zeta) \approx 0$. Thus **the Yukawa coupling "constant" of the dilaton-fermion interactions observable in gravitational experiments appears to be extremely strongly suppressed**[8]. The measure of the suppression can be characterized by using the constraint (21): the fermionic source in the dilaton equation (19), $F(\zeta)\overline{\Psi}'\Psi'$, is of the order of the dark energy density divided by M_p. This is the mechanism of the resolution of the fifth force problem in TMT. Notice that this mechanism does not suppress the interaction of the dilaton with the primordial fermion fields if the latter are not in the state corresponding to the first two generations.

Recall that the fermionic contribution to the energy-momentum tensor (15) consists of two terms: canonical and noncanonical. The latter is the dynamical fermionic $\Lambda^{(ferm)}_{dyn}$ term (proportional to $F(\zeta)$) which is of the order of the dark energy density. This is why *in gravitational experiments with observable matter the noncanonical part of the fermionic contribution to the energy-momentum tensor is negligible as compared to the canonical one.*

If the fermions of the third family would be a significant portion of the matter observable in gravitational experiments then anomalous gravitational effects could be detected. Indeed, the canonical part of the energy-momentum tensor for the third family fermion in the rest frame is

$$T^{(ferm,can)}_{\mu\nu}|_{3-rd\,family} = \delta_{\mu 0}\delta_{\nu 0}L^{(3-rd\,family)}_{selfint.} \tag{37}$$

whereas the correspondent noncanonical part reads

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[8]Note that the same conclusion can be obtained considering the dilaton field equation written in the form (22). Indeed, the values of ζ corresponding to the first two generations are constants and in particular they are $\overline{\Psi}'_q\Psi'_q$ independent with extremely high accuracy. Therefore substitution of the values of ζ into Eq.(22) in this case does not produce dilaton-fermion coupling.

$$T_{\mu\nu}^{(ferm,noncan)} = -\frac{1}{2}\tilde{g}_{\mu\nu}L_{selfint.}^{(3-rd\,family)}; \tag{38}$$

the latter is not negligible and could be the origin of gravitational anomalies.

It is interesting that although $F_i(\zeta) \neq 0$ for the third fermion family, its interaction with the dilaton turns out to be suppressed at the late-time universe. In fact, inserting ζ corresponding to the third fermion family, Eq.(34), into Eq.(22) we see that the coupling of the dilaton to the third fermion family includes the factor $e^{-2\alpha\phi/M_p}$ which is extremely small in the quintessential scenario of the late time universe. But this coupling might be important at earlier stages of the universe.

VIII. COSMO-LOW ENERGY PHYSICS STATES AND COSMOLOGY OF THE LATE-TIME UNIVERSE

A. The essence of the Cosmo-Particle Phenomena

1. Toy model

In the three previous sections we studied two opposite limiting cases: one is realized if there are no fermions at all or they appear only as test particles; the second one corresponds to the laboratory conditions of the normal particle physics where due to localizations, the magnitude of $|\overline{\Psi}'\Psi'|$ is so large that balance dictated by the constraint requires either $F_i(\zeta) \approx 0$ or $\zeta + b \approx 0$.

It turns out that besides these standard particle physics situations, TMT predicts possibility of so far unknown states which can be realized, for example, in astrophysics and cosmology. To illustrate some of the properties of such states let us start from a simplest (but non physical) model describing the following self-consistent system: the FRW universe filled by the homogeneous scalar field ϕ and a homogeneous primordial fermion field $\Psi' = \Psi'(t)$ (one can think for instance of a primordial neutrino) as sources of gravity. The fermion has zero momenta and therefore one can rewrite $\overline{\Psi}'\Psi'$ in the form of density $\overline{\Psi}'\Psi' = u^\dagger u$ where u is the large component of the Dirac spinor Ψ'. The space components of the 4-current $\tilde{e}_a^\mu \overline{\Psi}'\gamma^a\Psi'$ equal zero. It follows from the 4-current conservation that $\overline{\Psi}'\Psi' = u^\dagger u = \frac{const}{a^3}$ where $a = a(t)$ is the scale factor. It is convenient to rewrite the constraint (21) (adjusted to the model under consideration) in the form

$$\left\{\left[(\zeta - b)sM^4 e^{-2\alpha\phi/M_p} + V_1\right] + 2V_2\right\} + \frac{(\zeta + b)^{3/2}}{(\zeta + k)^2}[\zeta^2 + (3h_\nu - k)\zeta + 2b(h_\nu - k) + kh_\nu]\frac{const}{a^3} = 0, \tag{39}$$

where the function $F_\nu(\zeta)$, Eq.(18), has been used.

A possible solution of the constraint for the expanding universe as the scale factor $a(t) \to \infty$ is identical to the one studied in Sec.V where the fermion contribution is treated as negligible. There is however another solution where the decaying fermion contribution $u^\dagger u \sim \frac{const}{a^3}$ to the constraint is compensated by the appropriate behavior of the scalar field ζ. Namely if expansion of the universe is accompanied by approaching $\zeta \to -k$ then the contribution of the last term can be comparable with that of other terms at all times. This regime corresponds to a very unexpected state of the primordial fermion. First, this state does not belong to any generation of the regular fermions. Second, the effective mass of the fermion in this state increases like $(\zeta + k)^{-1}$ and the behavior of the dynamical fermionic $\Lambda_{dyn}^{(ferm)}$ term is $\Lambda_{dyn}^{(ferm)} \propto (\zeta + k)^{-2}u^\dagger u$. This means that at the late time universe, the canonical fermion energy density $\rho_{(ferm,canon)} \propto (\zeta + k)^{-1}u^\dagger u$ becomes much less than $\Lambda_{dyn}^{(ferm)}$. Third, such cold fermion matter possesses pressure and its equation of state in the late time universe approaches the form $p_{(ferm)} = -\rho_{(ferm)}$.

Since $\Lambda_{dyn}^{(ferm)}$ is of the order of the dark energy density, we conclude that for very low fermion energy densities (of the order of the dark energy density), TMT predicts a possibility of new type of states which we will refer as "Cosmo-Low Energy Physics" (CLEP) states. The effect of changing parameters (like a mass) of the fermion in CLEP state due to the described above its direct drawing into the cosmological expansion will be referred as **Cosmo-Particle Phenomenon**.

A more realistic model may be constructed by studying a possible effect of uniformly distributed neutrinos (and antineutrinos) on cosmological scenarios in the context of TMT. The simplest solution implies that primordial neutrino ν' is regarded as a free test fermion in the fermion vacuum (with ζ given by Eq.(23)) studied in Sec.V. Then the mass of the neutrino is very small and its asymptotic value (when assuming a quintessence-like scenario with $\phi \to \infty$ as $t \to \infty$) is determined by Eq.(30).

There is however a more interesting solution motivated by what was discussed above in the toy model. The important thing we have learned is a possibility of states where the primordial neutrinos and antineutrinos can be very heavy if $\zeta \approx -k$. This is why it may have sense to consider wave packets of free *non-relativistic neutrinos and antineutrinos*, i.e. with the momentum $|\vec{p}_\nu| \ll m_\nu$. We will assume also that the momentum-space widths of the wave packets are very narrow ($\Delta p_\nu \ll p_\nu$), which means that the wave packets differ not too much from, say, plane waves. Then $\overline{\nu'}\nu'$ are very small. We are going to study a solution when the small contribution of $\overline{\nu'}\nu'$ into the constraint

$$(\zeta - b)\left[sM^4 e^{-2\alpha\phi/M_p} + V_1\right] + 2V_2 + (\zeta + b)^2 F_\nu(\zeta)\overline{\nu'}\nu' = 0, \tag{40}$$

is compensated by large value of $F_\nu(\zeta)$ if $\zeta \approx -k$:

$$F_\nu(\zeta)|_{\zeta\approx-k} = \mu_\nu \frac{(h_\nu - k)(b - k)^{1/2}}{(\zeta + k)^2} + \mathcal{O}\left(\frac{1}{\zeta + k}\right). \tag{41}$$

A possible way to get up such a CLEP state might be spreading of the wave packets during their free motion lasting a very long (of the cosmological scale) time. One should note however that spreading is here much more complicated process than in the well known quantum mechanics text books example: decreasing of $\overline{\nu'}\nu'$ (due to the spreading) is related to the change of ζ which satisfies the nonlinear constraint equation. So we deal in this case with nonlinear quantum mechanics. The detailed study of the spreading of the wave packet in TMT is a subject of considerable interest but in this paper we will concentrate our attention on the consequences of the assumption that states with $\zeta \approx -k$ are achievable. We will see below that in the context of the cosmology of the late time universe such states can provide lower energy of the universe than the states with ζ determined in Sec.V where there are no fermions at all. In other words, independently of how the states with $\zeta \approx -k$ are realized they are energetically more preferable than states without massive fermionic matter at all.

It is clear that it is very hard to observe the primordial neutrinos in the CLEP states because $\overline{\nu'}\nu'$ is anomalously small. Furthermore, the attempt to detect the fermion in the CLEP state should be accompanied by its localization that destroy the condition for the existence of such states. This is the reason to believe that **primordial neutrinos and antineutrinos in the CLEP states might be good candidate for dark matter**.

B. Cosmology of the late time universe with the cold neutrino dark matter in the CLEP state

The study of the cosmology in the context of TMT becomes complicated in the presence of the fermionic matter because the averaging procedure of the matter and gravity equations of motion includes also the need to know the field ζ as a solution of the constraint. As it was noticed before (see discussion after Eq.(21)) this generically results in the appearance of a very nonlinear $\overline{\Psi'}\Psi'$-dependence in all equations of motion. This makes the averaging in the cosmological scales very unclear and practically unrealizable procedure.

Significant simplifications of the described general situation we observed in sections V and VIIIA were related to the fact that in those particular cases the function ζ appears to be only ϕ-dependent or constant (or approaching constant).

We are going now to apply the results of Sec.VIIIA to see whether it is possible to obtain a satisfactory scenario of the late time universe where the primordial neutrinos and antineutrinos in the CLEP states play the role of the cold dark matter. As the first step we will consider here a scenario where in addition to the homogeneous component of the scalar field ϕ, the universe is filled also by homogeneously distributed[9] cold neutrinos and antineutrinos in the CLEP states. We will assume that the cold neutrinos and antineutrinos are stable and their total number is stabilized

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[9]Actually one should know the clustering properties of the proposed cold dark matter model.

at the late time universe. Then after averaging over typical cosmological scales (resulting in the Hubble low), the constraint (21) reads

$$-(k+b)\left(sM^4 e^{-2\alpha\phi/M_p} + V_1\right) + 2V_2 + (b-k)^2 \frac{n_0^{(\nu)}}{a^3} F_\nu(\zeta)|_{\zeta\approx-k} = \mathcal{O}\left((\zeta+k)(sM^4 e^{-2\alpha\phi/M_p} + V_1)\right). \qquad (42)$$

where $F_\nu(\zeta)|_{\zeta\approx-k}$ is defined by Eq.(41) and $n_0^{(\nu)}$ is a constant determined by the total number of the cold neutrinos and antineutrinos.

Cosmological equations for a spatially flat FRW universe filled by the homogeneous scalar field ϕ and cold neutrino dark matter are the following

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left[\rho_{d.e.} + \rho_{d.m.}\right] \qquad (43)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{2\alpha k}{(b-k)^2 M_p}M^4 e^{-2\alpha\phi/M_p} + \mathcal{O}\left((\zeta+k)e^{-2\alpha\phi/M_p}\right) = 0 \qquad (44)$$

where the dark energy density $\rho_{d.e.}$ and dark matter energy density $\rho_{d.m.}$ are respectively

$$\rho_{d.e.}(\phi) = \frac{1}{2}\dot{\phi}^2 + \frac{bV_1 - V_2}{(b-k)^2} + \frac{b}{(b-k)^2}sM^4 e^{-2\alpha\phi/M_p} + \mathcal{O}(\zeta+k), \qquad (45)$$

$$\rho_{d.m.} = \left[\frac{\mu_\nu(h_\nu - k)}{(\zeta+k)(b-k)^{1/2}} - F_\nu(\zeta)|_{\zeta\approx-k}\right]\frac{n_0^{(\nu)}}{a^3} \qquad (46)$$

The constraint (42) and Eq.(41) show that the corrections $\mathcal{O}(\zeta+k)$ behave as

$$\mathcal{O}(\zeta+k) \propto \frac{1}{a^{3/2}} \qquad (47)$$

We will see that they are negligible if no fine tuning of the parameters of the theory will be done.

We choose $s = +1$ and in addition to Eq.(29) assume that

$$V_1 > 0, \quad V_2 > 0 \quad and \quad b > 0, \quad k < 0, \quad h_\nu < 0, \quad h_\nu - k < 0, \quad b + k < 0. \qquad (48)$$

In the case we work with a positive fermion energy solution, in order to provide positivity of the effective neutrino mass (see Eq.(20)) in the CLEP state, we should consider the regime where $\zeta = -k - \varepsilon$, $\varepsilon > 0$. Then $F_\nu(\zeta)|_{\zeta\approx-k} < 0$ and both of the terms in Eq.(46) are positive. The first term in Eq.(46) results from the canonical part $T_{00}^{(\nu,can)}$ of the neutrino energy-momentum tensor after making use of the equations for neutrino (antineutrino) field, neglecting the terms proportional to 3-momenta of the neutrinos and antineutrinos and averaging. The second term in Eq.(46) comes from the dynamical fermionic $\Lambda_{dyn}^{(ferm)}$ term.

The contribution of the cold neutrino dark matter to the pressure in the same approximation is determined only by the dynamical fermionic $\Lambda_{dyn}^{(ferm)}$ term and it is negative:

$$P_{d.m.} = \frac{n_0^{(\nu)}}{a^3}F_\nu(\zeta)|_{\zeta\approx-k} = -\frac{2V_2 + |b+k|V_1}{(b-k)^2} - \frac{|b+k|}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p} < 0. \qquad (49)$$

where the constraint (42) has been used.

Further, neglecting in (46) terms of the order of $(\zeta+k)^{-1}$ as compared to the terms of the order of $(\zeta+k)^{-2}$ and using again the constraint (42) one can rewrite $\rho_{d.m.}$ in the form

$$\rho_{d.m.} = \frac{2V_2 + |b+k|V_1}{(b-k)^2} + \frac{|b+k|}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p} \qquad (50)$$

typical for the dark energy sector including both a cosmological constant and a potential (compare (50) with (45)). The accuracy of this approximation grows as $a(t) \to \infty$.

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The very important feature of **the cold dark matter** realized via the CLEP states is that it **possesses negative pressure** and in the context of a quintessence-like scenario (i.e. $\phi \to \infty$ as $a(t) \to \infty$) of the FRW cosmology, its equation of state approaches that of the cosmological constant ($P_{d.m.} = -\rho_{d.m.}$) as $a(t) \to \infty$. In the same asymptotic regime, the ratio $\Omega_{d.e.}/\Omega_{d.m.}$ approach the constant

$$\frac{\Omega_{d.e.}}{\Omega_{d.m.}} \to \frac{bV_1 - V_2}{|b+k|V_1 + 2V_2} \quad as \quad a(t) \to \infty \tag{51}$$

that solves (at least qualitatively) the problem of the cosmic coincidence. Note that there is a broad range of the parameters which allow fitting of the desirable value of $\frac{\Omega_{d.e.}}{\Omega_{d.m.}} \approx 2$. This value is achieved if we assume for example that $b = \frac{2}{3}|k|$ and $V_2 \ll bV_1$.

The reason we regard our dark matter model as the cold one is the imposed condition that neutrinos and antineutrinos are nonrelativistic. The latter becomes possible because the fermion mass in CLEP state increases as $m_\nu(\zeta) \propto (\zeta + k)^{-1} \propto a^{3/2}$. We would like to emphasize once more that the strange enough fact, that the CLEP-neutrino cold matter possesses negative pressure, is possible due to the presence of the noncanonical part of the fermion energy-momentum tensor, Eq.(17), having the form of a dynamical fermionic $\Lambda_{dyn}^{(ferm)}$ term.

The total energy density and the total pressure of the dark sector (including both dark energy and dark matter) in the framework of the explained above approximations can be represented in the form

$$\rho_{dark}^{(total)} \equiv \rho_{d.e} + \rho_{d.m} = \frac{1}{2}\dot{\phi}^2 + U_{dark}^{(total)}(\phi); \quad P_{dark}^{(total)} \equiv P_{d.e.} + P_{d.m.} = \frac{1}{2}\dot{\phi}^2 - U_{dark}^{(total)}(\phi), \tag{52}$$

where the effective potential $U_{dark}^{(total)}(\phi)$ is the sum

$$U_{dark}^{(total)}(\phi) \equiv \Lambda + V_{quint}(\phi), \tag{53}$$

of the cosmological constant

$$\Lambda = \frac{V_2 + |k|V_1}{(b-k)^2} \tag{54}$$

and the exponential potential

$$V_{quint}(\phi) = \frac{|k|}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p}. \tag{55}$$

This means that the evolution of the late time universe in the state with $\zeta \approx -k$ proceeds as it would be in the standard field theory model (non-TMT) including **both the cosmological constant and the quintessence field ϕ with the exponential potential**[10]. The emergence of the cosmological constant guarantees an accelerated expansion of the universe. This is why, in contrast with the standard quintessence scenario with exponential potential, we should not worry here about the value of the parameter α in order to provide accelerating expansion[11]. Note that to provide the observable energy densities (for example, $\Lambda \sim \rho_{crit}$, where ρ_{crit} is the present day critical energy density) there is no need of fine tuning of the dimensionfull parameters V_1 and V_2 but instead one can assume that the dimensionless parameters b, k are very large.

Summarizing what we have learned in this section and in Sec.V, we conclude that in order to satisfy the constraint for very small $|\overline{\Psi}'\Psi'|$ one can proceed only in two possible ways:
i) to ignore the fermionic contribution to the constraint and solve it for ζ in a state "absent of fermions" or, alternatively
ii) to study the CLEP states ($\zeta \to -k$) where the fermionic contribution is not negligible.

It is very interesting to compare the effective potential $V_{eff}^{(0)}(\phi)$, Eq.(24), predicted for the universe filled only by the homogeneous scalar field (for short, a state "absent of fermions"), on the one hand, with the effective dark sector

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[10]Models with dark energy including both the cosmological constant and the quintessence field has been discussed recently [43]

[11]The model with $V_1 = V_2 = 0$ is shortly presented in Appendix D where the appropriate estimations for the asymptotic ratio $\Omega_{d.e.}/\Omega_{d.m.}$ are made.

potential $U_{dark}^{(total)}(\phi)$ for the universe filled both by the homogeneous scalar field and by the homogeneous cold dark neutrino matter (for short, "CLEP state"), on the other hand. The **remarkable result** consists in the fact that if $bV_1 > V_2$, which is needed for positivity of $\Lambda^{(0)}$ (see Eqs.(26),(29)), then

$$V_{eff}^{(0)}(\phi) - U_{dark}^{(total)}(\phi) \equiv \frac{\left[\frac{b+k}{2}\left(V_1 + M^4 e^{-2\alpha\phi/M_p}\right) - V_2\right]^2}{4(b-k)^2\left[b\left(V_1 + M^4 e^{-2\alpha\phi/M_p}\right) - V_2\right]} > 0. \tag{56}$$

This means that (for the same value of $\dot{\phi}^2$) **the universe in "the CLEP state" has a lower energy density than the one in the "absent of fermions" state**. One should emphasize that this result does not imply at all that $\rho_{d.m}$ is negative.

For illustration of what kind of solutions one can expect, let us take the *particular value* for the parameter α, namely $\alpha = \sqrt{3/8}$. Then in the framework of the explained above approximations, the cosmological equations allow the following analytic solution:

$$\phi(t) = \frac{M_p}{2\alpha}\varphi_0 + \frac{M_p}{2\alpha}\ln(M_p t), \qquad a(t) \propto t^{1/3} e^{\lambda t}, \tag{57}$$

where

$$\lambda = \frac{1}{M_p}\sqrt{\frac{\Lambda}{3}}, \qquad e^{-\varphi_0} = \frac{2(b-k)^2 M_p^2}{\sqrt{3}|k|M^4}\sqrt{\Lambda}. \tag{58}$$

and Λ is determined by Eq.(54). The mass of the neutrino in such CLEP state increases exponentially in time and its ϕ dependence is double-exponential:

$$m_\nu|_{CLEP} \sim (\zeta + k)^{-1} \sim a^{3/2}(t) \sim t^{1/2} e^{\frac{3}{2}\lambda t} \sim \exp\left[\frac{3\lambda e^{-\varphi_0}}{2M_p}\exp\left(\frac{2\alpha}{M_p}\phi\right)\right]. \tag{59}$$

In this particular case, the time dependence of the ratio $\Omega_{d.e.}/\Omega_{d.m.}$ reads

$$\frac{\Omega_{d.e.}}{\Omega_{d.m.}} \approx \frac{bV_1 - V_2 + \frac{2}{3|k|}b(b+|k|)(V_2 + |k|V_1)^{1/2} \cdot \frac{M_p}{t}}{|b+k|V_1 + 2V_2 + \frac{2}{3|k|}(k^2 - b^2)(V_2 + |k|V_1)^{1/2} \cdot \frac{M_p}{t}}, \tag{60}$$

where we have neglected corrections of the order t^{-2} coming from the kinetic term of the scalar field ϕ.

IX. DISCUSSION AND CONCLUSION

A. Summary and some possible directions of possible developments

In this paper we have continued our study of the two measures theory, including fermions. Except for three special physical requirements, the theory starts from the most general action which possesses global scale symmetry (12) and contains one primordial fermion field for each type of leptons and quarks, i.e. does not incorporate fermion generations. It turns out that each of the primordial fermion fields can be in different states depending on the fermion density. The states of the primordial fermion are controlled by the constraint (21) which appears as the consistency condition of the classical equations of motion.

In the regime of the "high fermion energy density" which is always realized in the normal particle, nuclear and atomic physics experiments, the theory predicts splitting of each type of the leptons and quarks exactly into three mass eigenstates. So, TMT appears to be able to provide the resolution of the fermion families problem. Under the same "high fermion energy density" conditions, the fermion-dilaton coupling suffers an extremely high suppression that means that the fermion families problem is resolved simultaneously with the fifth force problem.

If the fermion energy density is of the order of dark energy density (which we call the regime of low fermion energy density) it has been shown in this paper that the primordial fermion does not split into generations and also it can be in a CLEP state where it participates directly in the cosmological expansion by means of changing its own parameters (like a mass).

The explicit solution of equations of motion was presented where uniformly distributed nonrelativistic neutrinos and antineutrinos in CLEP state appear as a good model for cold dark matter scaling like a dark energy. The total

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equation of state provides accelerated expansion of the universe. Another attractive feature of this scenario is that the total energy density of such universe is less than that in the case of the universe free of the fermions at all.

Together with the constraint (21), the key role both in the "high fermion energy density" (regular fermion families and resolution of the fifth force problem) and in the low fermion energy density (cosmo-particle solution) belongs to the dynamical fermionic $\Lambda_{dyn}^{(ferm)}$ term, Eq(17). The constraint dictates that if ζ is $\overline{\Psi'}\Psi'$ independent (which is true in all problems studied in the present paper except for the case of the third fermion family solution), then the value of $\Lambda_{dyn}^{(ferm)}$ must be of the order of the dark energy density. If the canonical fermion energy density is much larger than $\Lambda_{dyn}^{(ferm)}$ then we deal with the "high fermion energy density" case where the primordial fermions are in the mass eigenstates of the first two families of the regular fermion matter. However, if the canonical fermion energy density is much less than $\Lambda_{dyn}^{(ferm)}$ which we call the low fermion energy density regime, then besides the homogeneous cosmological solutions studied in this paper, there are many inhomogeneous regimes of great interest from the astrophysical point of view. The important feature of the theory is that the constraint is a local algebraic equation and therefore it dictates not only the same scaling in time for fermion and dark energy density at the late time universe but also the space fluctuations of the fermion and dilaton energy densities turn out to be strongly correlated in the low fermion energy density regime. Besides, in this regime there is no suppression of the fermion-dilaton coupling and therefore the fermion (in our scenario, neutrino) dark matter should be self-interacting via scalar force. Taking into account also that the (primordial) fermionic matter in this regime has negative pressure[12], one should expect the emergence of new types of mechanisms for clumping of dark matter and structure formation. These intriguing questions should be the subject of future investigations.

B. Similarities with ideas and/or results discussed by other authors

It is interesting to notice that in a certain sense the theory being studied here (where nonlinear effects are found at very low energy densities) resembles the nonlinear theory at very low accelerations (MOND) [45]. The details are quite different, but fact remains that nonlinear effects make the difference (i.e. the appearance of dark matter or dark matter effects) in both cases just in the weak limit.

The theory discussed here displays some of the features associated to the Chaplygin gas models [46]. In particular it is known that such "gas" behaves like normal matter (dust) at high densities and like vacuum energy at low densities. Also this takes place in our TMT model where normal particle states emerge at high fermion densities and for low fermion densities we have shown the universe prefers to be in a state where the contribution of fermionic matter is asymptotically dominated by the dynamical fermion Λ term.

In this paper we have found that particles are able to react to the cosmological evolution and change their masses, according to the constraint equation. This constraint in turn implies a cosmic coincidence. All this is realized in a way that very much resembles the steady state model [47], discarded as a description of the whole history of the universe, but which in our case it seems to reappear as a good approximation for the asymptotic behavior of the late universe. Here the "continuous matter creation" [47] is replaced not only by the variable particle masses (similar to VAMPS models) but rather by a much more important effect related with the role of $\Lambda_{dyn}^{(ferm)}$ term which is originated by fermionic (neutrino) matter and behaves as a cosmological "constant". In the theory studied here, the coincidence of dark energy and dark matter densities can be achieved in a way we consider where the asymptotic values of the dark energy and dark matter densities are constants. This indeed strengthens the similarity between our asymptotic state and the steady state models.

We should point out the interesting convergence between some of our results and those of the authors of Ref. [48], where also the possibility of mass varying neutrinos was considered. Although their starting point is rather different, many of their results actually are qualitatively similar to ours: (i) the neutrino can behave as a negative pressure fluid; (ii) the neutrino mass is correlated to the neutrino density, etc..

Finally, we find that both in the case of high fermion density and for low fermion density, the presence of the fermions changes drastically the vacuum in which the fermions live. The effects here are rather dramatic: in the high density it is responsible for the three generations splitting of the primordial fermions and at low densities - for the cosmic coincidence between dark energy and dark matter densities. For more conventional type of field theories, the question of whether a fermion can change the properties of the vacuum in which it lives, was studied in Ref. [49] and

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[12]The negative pressure is needed apparently for explaining the flat rotation curves (see for example [44])

found that this can be indeed the case. The authors of Ref. [49] have demonstrated this by obtaining a substantially different fermion mass in the self-consistent approach as compared to the one obtained if one had considered such a fermion as a simple "test particle".

X. ACKNOWLEDGEMENTS

APPENDIX A: POSSIBLE ORIGIN OF TMT FROM LOW ENERGY LIMIT OF THE BRANE-WORLD SCENARIO

Continuing discussion of the general structure of the action in TMT one of course may ask: why 3-brane moving only in $3+1$ dimensional embedding space-time? This can be obtained also starting from higher dimensional "brane-world scenarios". Indeed, let us consider for example a 3-brane evolving in an embedding 5-dimensional space-time with

$$ds^2 = G_{AB}dx^A dx^B = f(y)\hat{g}_{\mu\nu}(x^\alpha)dx^\mu dx^\nu + \gamma^2(y)dy^2, \quad A, B = 0, 1, ...4; \quad \mu, \nu = 0, 1, 2, 3. \tag{A1}$$

Assuming that it is possible to ignore the motion of the brane in the extra dimension, i.e. studying the brane with a fixed position in extra dimension, $y = const$, one can repeat the arguments of Sec.II (starting with the Nambu-Goto action) where one needs to use only four functions $\varphi^a(x^\mu)$ which together with the fifth (x^μ independent) component along the axis y constitute the 5-vector describing the embedding of our brane; L_1 is again (as in Sec.II) the φ^a independent Lagrangian of the gravity[13] and matter on the brane. As a result we obtain exactly the same effective action as in Eq.(9) which describes a brane moving in the hypersurface $y = const$ of the 5-dimensional space-time.

The brane theory action contains also a piece coming from the bulk dynamics. We will assume that gravity and matter exist also in the bulk where their action can be written in the form

$$S_{bulk} = \int \sqrt{-\hat{g}} f^2(y)\gamma(y) L_{bulk} d^4x dy \tag{A2}$$

where $\hat{g} \equiv det(\hat{g}_{\mu\nu})$ and L_{bulk} is the Lagrangian of the gravity and matter in the bulk. We are not interested in the dynamics in the hole bulk but rather in the effect of the bulk on the 4-dimensional dynamics. For this purpose one can integrate out the perpendicular coordinate y in the action (A2). One can think of this integration in a spirit of the procedure known as averaging (for the case of compact extra dimensions see for example Ref. [51]). We do not perform this integration explicitly[14] here but we expect that the resulting averaged contribution of the bulk dynamics to the 4-dimensional action one can write down in the form

$$S_2 = \int \sqrt{-\hat{g}} L_2(\hat{g}_{\mu\nu}, connection, matter fields) d^4x \tag{A3}$$

that we would like to use as the second term of the postulated in Eq.(1) general form of the TMT action in four dimensions.

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[13]Notice that connection coefficients of our 3-brane are those where indexes run only from zero to three and these components do not suffer from discontinuities across the brane [50]

[14]The correspondent calculations must take into account the discontinuity constraints.

Notice however that the geometrical objects in these two actions may not be identical. The simplest assumption would be of course to take $\hat{g}_{\mu\nu} \equiv g_{\mu\nu}$ (and also coinciding connections). One may allow for the case $\hat{g}_{\mu\nu} \neq g_{\mu\nu}$ nevertheless. In fact, brane theory allows naturally bimetric theories [52] even if one starts with a single bulk metric. In our case this can be due to the fact that the metric at the brane and its average value may be different. The bimetric theories [53] give in any case only one massless linear combination of the two metrics which one can identify as long distance gravity. Connections at the brane and its average value may be different as well. But we expect this difference to be small due to the continuity of the relevant connection coefficients (see footnote 11). Therefore performing the integration over extra dimension y we are left with just one independent connection which is important for TMT where the first order formalism is supposed to be one of the basic principles. All these arguments are now being developed in details and will be reported elsewhere.

APPENDIX B: EQUATIONS OF MOTION AND CONSTRAINT IN THE ORIGINAL FRAME

Equation (5) corresponding to the model (10) reads:

$$\left[-\frac{1}{\kappa}R(\omega,e) + \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]e^{\alpha\phi/M_p} - V_1 e^{2\alpha\phi/M_p} + L_{fk}e^{\alpha\phi/M_p} - \sum_{i=\nu,E}\mu_i\overline{\Psi}_i\Psi_i e^{\frac{3}{2}\alpha\phi/M_p} = sM^4, \tag{B1}$$

where

$$L_{fk} = \frac{i}{2}\left[\overline{\nu}\left(\gamma^a e_a^\mu\overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu\gamma^a e_a^\mu\right)\nu + \overline{E}\left(\gamma^a e_a^\mu\overrightarrow{\nabla}_\mu^{(E)} - \overleftarrow{\nabla}_\mu^{(E)}\gamma^a e_a^\mu\right)E\right]. \tag{B2}$$

Variation of the action (10) with respect to $e^{a\mu}$ yields

$$(\zeta + b)\left[-\frac{2}{\kappa}R_{a,\mu}(\omega,e) + e_a^\beta\phi_{,\mu}\phi_{,\beta}\right]$$

$$+ g_{\mu\beta}e_a^\beta\left[\frac{b}{\kappa}R(\omega,e) - \frac{b}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + V_2 e^{\alpha\phi/M_p} - kL_{fk} + (h_\nu\mu_\nu\overline{\nu}\nu + h_E\mu_E\overline{E}E)e^{\frac{1}{2}\alpha\phi/M_p}\right]$$

$$+ (\zeta + b)\frac{i}{2}\left[\overline{\nu}\left(\gamma^a\overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu\gamma^a\right)\nu + \overline{E}\left(\gamma^a\overrightarrow{\nabla}_\mu^{(E)} - \overleftarrow{\nabla}_\mu^{(E)}\gamma^a\right)E\right] + \frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}L_{em}}{\partial e^{a,\mu}}e^{-\alpha\phi/M_p} = 0 \tag{B3}$$

where $L_{em} = -\frac{1}{4}g^{\alpha\beta}g^{\mu\nu}F_{\alpha\mu}F_{\beta\nu}$.

Contraction of Eq.(B3) with $e^{a,\mu}$ gives

$$2(\zeta - b)\left[-\frac{1}{\kappa}R(\omega,e) + \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right] + 4V_2 e^{\alpha\phi/M_p} + (\zeta - 3k)L_{fk} + 4(h_\nu\mu_\nu\overline{\nu}\nu + h_E\mu_E\overline{E}E)e^{\frac{1}{2}\alpha\phi/M_p} = 0, \tag{B4}$$

where identity $e^{a\mu}(\partial\sqrt{-g}L_{em}/\partial e^{a\mu}) \equiv 0$ has been used.

Excluding $R(\omega,e)$ from Eqs.(B2) and (B4) we obtain consistency condition of these two equations:

$$2(\zeta - b)\left(sM^4 e^{-\alpha\phi/M_p} + V_1 e^{\alpha\phi/M_p}\right) + 4V_2 e^{\alpha\phi/M_p} - (\zeta - 2b + 3k)L_{fk}$$

$$+ 2[(\zeta - b + 2h_\nu)\mu_\nu\overline{\nu}\nu + (\zeta - b + 2h_E)\mu_E\overline{E}E)]e^{\frac{1}{2}\alpha\phi/M_p} = 0, \tag{B5}$$

Varying $\overline{\nu}$ and ν we get:

$$(\zeta + k)\frac{i}{2}\left[\gamma^a e_a^\beta(\overrightarrow{\partial}_\beta + \frac{1}{2}\omega_\beta^{cd}\sigma_{cd}) + \frac{1}{2}\omega_\beta^{cd}\sigma_{cd}\gamma^a e_a^\beta\right]\nu$$

$$+ \frac{i}{2\sqrt{-g}}e^{-\alpha\phi/M_p}\partial_\beta\left[\sqrt{-g}(\zeta + k)e^{\alpha\phi/M_p}\gamma^a e_a^\beta\nu\right] - (\zeta + h_\nu)\mu_\nu\nu e^{\frac{1}{2}\alpha\phi/M_p} = 0 \tag{B6}$$

$$(\zeta + k)\frac{i}{2}\left[-\overline{\nu}(\overleftarrow{\partial}_\beta - \frac{1}{2}\omega_\beta^{cd}\sigma_{cd})\gamma^a e_a^\alpha + \overline{\nu}\gamma^a e_a^\beta\frac{1}{2}\omega_\beta^{cd}\sigma_{cd}\right]$$

$$- \frac{i}{2\sqrt{-g}}e^{-\alpha\phi/M_p}\partial_\beta\left[\sqrt{-g}(\zeta + k)e^{\alpha\phi/M_p}\overline{\nu}\gamma^a e_a^\beta\right] - (\zeta + h_\nu)\mu_\nu\overline{\nu}e^{\frac{1}{2}\alpha\phi/M_p} = 0 \tag{B7}$$

and similarly for E and $\overline{E}$. Multiplying the last two equations by $\overline{\nu}$ and ν respectively (and similarly for E and $\overline{E}$) we obtain

$$L_{fk} = \frac{1}{(\zeta + k)}[(\zeta + h_\nu)\mu_\nu\overline{\nu}\nu + (\zeta + h_E)\mu_E\overline{E}E]e^{\frac{1}{2}\alpha\phi/M_p} \tag{B8}$$

Inserting L_{fk} into the consistency condition, Eq.(B5), we get *the constraint (in the original frame)* as the following algebraic equation

$$(\zeta - b)\left(sM^4 e^{-2\alpha\phi/M_p} + V_1\right) + 2V_2 + \frac{e^{\frac{1}{2}\alpha\phi/M_p}}{2(\zeta + k)}\sum_{i=\nu,E} f_i(\zeta)\mu_i\overline{\Psi}_i\Psi_i = 0. \tag{B9}$$

where

$$f_i(\zeta) \equiv \zeta^2 + (3h_i - k)\zeta + 2b(h_i - k) + kh_i, \quad i = \nu, E \tag{B10}$$

Contracting Eq.(B3) with factor e^a_ν and using Eq.(B8) we get the gravitational equations in the original frame

$$\frac{2}{\kappa}R_{\mu\nu}(\omega, e) = \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\frac{1}{\zeta + b}\left(bsM^4 e^{-\alpha\phi/M_p} + (bV_1 - V_2)e^{\alpha\phi/M_p}\right)$$
$$+ \frac{\zeta + k}{\zeta + b}\frac{i}{2}\sum_{i=\nu,E}\overline{\Psi}_i\left(\gamma^a e_{a\nu}\overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu\gamma^a e_{a\nu}\right)\Psi_i + g_{\mu\nu}\frac{1}{\zeta + k}e^{\frac{1}{2}\alpha\phi/M_p}\sum_{i=\nu,E}(h_i - k)\mu_i\overline{\Psi}_i\Psi_i \tag{B11}$$

where for short we have omitted term with $F_{\mu\nu}$ since it is clear that due to the conformal invariance of L_{em} its contribution to the energy-momentum tensor will be canonical in the Einstein frame.

The scalar field ϕ equation of motion in the original frame can be written in the form

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[e^{\alpha\phi/M_p}(\zeta + b)\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right]$$
$$-\frac{\alpha}{M_p}\left[(\zeta + b)sM^4 + ((b - \zeta)V_1 - 2V_2)e^{2\alpha\phi/M_p} - \frac{e^{\frac{3}{2}\alpha\phi/M_p}}{2(\zeta + k)}\sum_{i=\nu,E}f_i(\zeta)\mu_i\overline{\Psi}_i\Psi_i\right] \tag{B12}$$

where Eqs.(B1), (B8) and the notation (B10) have been used.

APPENDIX C: CONNECTION IN THE ORIGINAL FRAME

We present here what is the dependence of the spin connection ω_μ^{ab} on e_μ^a, ζ, Ψ and $\overline{\Psi}$. Varying the action (10) with respect to ω_μ^{ab} and making use that

$$R(V, \omega) \equiv -\frac{1}{4\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}e_\alpha^c e_\beta^d R_{\mu\nu}^{ab}(\omega) \tag{C1}$$

we obtain

$$\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}[(\zeta + b)e_\alpha^c D_\nu e_\beta^d + \frac{1}{2}e_\alpha^c v_\beta^d\zeta_{,\nu}] + \frac{\kappa}{4}\sqrt{-g}\frac{\zeta + k}{\zeta + b}e^{c\mu}\varepsilon_{abcd}\overline{\Psi}\gamma^5\gamma^d\Psi = 0, \tag{C2}$$

where

$$D_\nu e_{a\beta} \equiv \partial_\nu e_{a\beta} + \omega_{\nu a}^d e_{d\beta} \tag{C3}$$

The solution of Eq. (C2) is represented in the form

$$\omega_\mu^{ab} = \omega_\mu^{ab}(e) + K_\mu^{ab}(e, \overline{\Psi}, \Psi) + K_\mu^{ab}(\zeta) \tag{C4}$$

where

$$\omega_\mu^{ab}(e) = e_\alpha^a e^{b\nu}\{_{\mu\nu}^\alpha\} - e^{b\nu}\partial_\mu e_\nu^a \tag{C5}$$

is the Riemannian part of the spin-connection,

$$K_\mu^{ab}(e,\overline{\Psi},\Psi) = \frac{\kappa}{8}\frac{\zeta+k}{\zeta+b}\eta_{cn}e_{d\mu}\varepsilon^{abcd}\overline{\Psi}\gamma^5\gamma^n\Psi \tag{C6}$$

is the fermionic contribution [54] and

$$K_\mu^{ab}(\zeta) = \frac{1}{2(\zeta+b)}\zeta_{,\alpha}(e_\mu^a e^{b\alpha} - e_\mu^b e^{a\alpha}) \tag{C7}$$

is **the non-Riemannian part of the spin-connection originated by specific features of TMT**.

The term $K_\mu^{ab}(e,\overline{\Psi},\Psi)$ is responsible for a correction to the Einstein equations (in the Einstein frame) having the form of the so-called spin-spin contact interaction [54] and proportional to the square of the Newton constant. Notice also that in the context of homogeneous cosmology this contribution is absent at all. If some inhomogeneities present, this contribution has additional suppression in the regime $\zeta \approx -k$.

APPENDIX D: A MODEL WITHOUT EXPLICIT POTENTIALS

In the recent paper [28] we have studied the model similar to one of the present paper but without explicit potentials in the action ($V_1 = V_2 = 0$ in Eq.(10)). In this case the effective exponential potential is generated by the spontaneous symmetry breaking discussed in Sections II and IV of the present paper. The aim of this Appendix is to check whether it is possible, in the context of the model described by the action (10) where now we take $V_1 = V_2 = 0$, to provide conditions both for an accelerated expansion and for a resolution of the cosmic coincidence problem in a way similar to what we have seen Sec.VIII, that is in the framework of a scenario with the cold neutrino dark matter in CLEP state.

Equations of motion and constraint of the model have the form of equations of Sec.IV where now $V_1 = V_2 = 0$. For short we will not write down the new equations here. Instead, in this Appendix we will refer to equations of motion and constraint of Sec.IV keeping in mind that now we have $V_1 = V_2 = 0$.

Let us start from a simple notion that in the case of the absence of fermions, i.e. when we deal with a spatially flat FRW universe filled only by the homogeneous scalar field ϕ, the constraint (21) yields $\zeta = b$. Therefore we have in this case a standard quintessence model with the exponential potential

$$V_{eff}^{(0)}(\phi) = \frac{M^4}{4b}e^{-2\alpha\phi/M_p}. \tag{D1}$$

Turn now to a spatially flat FRW universe filled by the homogeneous quintessence field ϕ and cold neutrino dark matter in the CLEP state. The respective cosmological equations read (we choose the same restrictions on the dimensionless parameters as in Eq.(48))

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}[\rho_q + \rho_{d.m.}] \tag{D2}$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{2\alpha|k|}{(b-k)^2 M_p}M^4 e^{-2\alpha\phi/M_p} + \mathcal{O}\left((\zeta+k)e^{-2\alpha\phi/M_p}\right) = 0, \tag{D3}$$

where the quintessence energy density is

$$\rho_q(\phi) = \frac{1}{2}\dot{\phi}^2 + \frac{b}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p} + \mathcal{O}\left((\zeta+k)e^{-2\alpha\phi/M_p}\right) \tag{D4}$$

and the dark matter energy density has the same form as in Eq.(46) with the function $F_\nu(\zeta)|_{\zeta\approx-k}$ determined by Eq.(41).

The constraint that reads now

$$-(k+b)M^4 e^{-2\alpha\phi/M_p} + (b-k)^2 \frac{n_0^{(\nu)}}{a^3} F_\nu(\zeta)|_{\zeta\approx-k} = \mathcal{O}\left((\zeta+k)M^4 e^{-2\alpha\phi/M_p}\right) \tag{D5}$$

allows to represent the dark matter energy density as the function of the quintessence field:

$$\rho_{d.m.} = \frac{(b+k)}{(b-k)^3}M^4(\zeta+k)e^{-2\alpha\phi/M_p} - \frac{b+k}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p} + \mathcal{O}\left(\frac{1}{(\zeta+k)a^3}\right). \tag{D6}$$

It follows from the constraint (D5) that behaviour both of the first term in (D6) and of the corrections $\mathcal{O}\left(\frac{1}{(\zeta+k)a^3}\right)$ are of the form

$$(\zeta+k)e^{-2\alpha\phi/M_p} \propto \mathcal{O}\left(\frac{1}{(\zeta+k)a^3}\right) \propto \frac{e^{-\alpha\phi/M_p}}{a^{3/2}}. \tag{D7}$$

The same order of corrections have appeared both in Eq.(D2) and in Eq.(D3). As we will see below, if *one to demand* that solutions of the cosmological equations describe *an accelerated expansion* of the universe, this is sufficient to provide that all such corrections decrease more rapidly than the rest of the terms. This is why studying the late time (accelerated) universe one can regard all these corrections as negligible.

Ignoring the above mentioned corrections, we have a situation where the dark matter contribution to the energy-momentum tensor behaves as an additional exponential potential and the dark matter equation of state approaches that of the cosmological constant ($P_{d.m.} = -\rho_{d.m.}$) as $a(t) \to \infty$. As a result of this, the total energy density and total pressure are in this approximation:

$$\rho_{tot} \equiv \rho_q + \rho_{d.m.} = \frac{1}{2}\dot{\phi}^2 + \frac{|k|}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p} \tag{D8}$$

$$P_{tot} \equiv P_q + P_{d.m.} = \frac{1}{2}\dot{\phi}^2 - \frac{|k|}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p} \tag{D9}$$

This means that in the late time universe the cosmological equations (D2), (D3) are reduced to the standard ones of the quintessential cosmology with the exponential potential $\frac{|k|}{(b-k)^2}M^4 e^{-2\alpha\phi/M_p}$. It is easy to show that similar to what we have seen in the model of Sec.VIIIB, Eq.(56), *the universe in "the CLEP state" has a lower energy density than the one in the "absent of fermions" state.*

The correspondent cosmological solution is the following:

$$\phi(t) = \frac{M_p}{2\alpha}\varphi_0 + \frac{M_p}{\alpha}\ln(M_p t), \qquad a(t) \propto (M_p t)^{1/2\alpha^2}, \qquad e^{-\phi_0} = \frac{(3-2\alpha^2)(b-k)^2 M_p^4}{4\alpha^4 |k| M^4}. \tag{D10}$$

According to this solution, the universe expands with acceleration if $\alpha < \frac{1}{\sqrt{2}}$. In such a case the corrections we ignored solving the above cosmological equations, behave in time as

$$\frac{1}{(\zeta+k)a^3} \propto (\zeta+k)e^{-2\alpha\phi/M_p} \propto \frac{e^{-\alpha\phi/M_p}}{a^{3/2}} \propto t^{-(1+3/4\alpha^2)} < \frac{1}{t^{5/2}}. \tag{D11}$$

The time dependence of the quintessence and dark matter energy densities corresponding to the solution (D10) are respectively:

$$\rho_q = \frac{1}{4\alpha^4 |k|}[2(|k|-b)\alpha^2 + 3b] \cdot \frac{M_p^2}{t^2}, \tag{D12}$$

$$\rho_{d.m.} = \frac{1}{4\alpha^4 |k|}(|k|-b)(3-2\alpha^2) \cdot \frac{M_p^2}{t^2} \tag{D13}$$

Then for the ratio

$$\frac{\Omega_q}{\Omega_{d.m.}} = \frac{2(|k| - b)\alpha^2 + 3b}{(|k| - b)(3 - 2\alpha^2)} \tag{D14}$$

in the case of accelerated expansion $(\alpha^2 < 1/2)$ we obtain

$$\frac{\Omega_q}{\Omega_{d.m.}} < \frac{|k| + 2b}{2(|k| - b)}. \tag{D15}$$

So, we conclude that the model without explicit potentials provides conditions for a cosmic coincidence in the scenario of the accelerated expansion of the late time universe.

The time dependence of the mass of the neutrino in CLEP state is

$$m_\nu|_{CLEP} \sim t^{-1+3/4\alpha^2} \tag{D16}$$

and in the scenario of an accelerating expansion it increases in time faster than $t^{1/2}$.

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[1] A. G. Riess et al., Astron. J. **116**, 1009 (1998); S. Perlmutter et al., Astrophys. J. **517**, 565 (1999); N. Bahcall, J.P. Ostriker, S.J. Perlmutter and P.J. Steinhardt, Science **284**, 1481 (1999).

[2] D.N. Spergel et al., Astrophys. J. Supp. **148**, 175 (2003), astro-ph/0302209.

[3] S.M.Carroll, Living Rev. Rel. **4**, 1 (2001); P.J.E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); astro-ph/0207347; T. Padmanabhan, Phys. Rep. **380**, 235 (2003) (hep-th/0212290).

[4] J.R. Ellis, astro-ph/0304183; J.R. Primack, Nucl.Phys.Proc.Suppl.**124**, 3 (2003) ; astro-ph/0205391

[5] C. Wetterich, Nucl. Phys. **B302**, 668 (1988). B. Ratra and P.J.E. Peebles , Phys. Rev. **D37**, 3406 (1988); P.J.E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988). R. Caldwell, R. Dave and P. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); N. Weiss, Phys. Lett. **B197**, 42 (1987); Y. Fujii and T. Nishioka, Phys. Rev. **D42**, 361 (1990); M.S. Turner and M. White, Phys. Rev. **D56**, R4439 (1997); P. Ferreira and M.Joyce, Phys. Rev. Lett. **79**, 4740 (1997); Phys. Rev. **D58**, 023503 (1998). E. Copeland, A. Liddle and D. Wands, Phys. Rev. D57, 4686 (1998); P. Steinhardt, L Wang and I. Zlatev, Phys. Rev. **D59**, 123504 (1999);

[6] I. Zlatev, L Wang and P. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999).

[7] C. Wetterich, Nucl. Phys. **B302**, 645 (1988); Astron. Astrophys. **301**, 321 (1995); D. Wands, E.S. Copeland and A. Liddle, Ann. N.Y. Acad. Sci. **688**, 647 (1993).

[8] E.I. Guendelman, Phys.Lett. B**193**, 433 (1987); ibid., B**201**, 39 (1988).

[9] J.A.Casas, J. Garcia-Bellido and M. Quiros, Class. Quant. Grav. **9** 1371 (1992); G.W. Anderson and S.M. Carroll, astro-ph/9711288.

[10] L. Amendola, Phys. Rev. D**62**, 043511 (2000)

[11] D.J. Holden and D. Wands, Phys. Rev. D**61**, 043506 (2000); A.P. Billyard and A.A. Coley, Phys. Rev. D**61**, 083503 (2000); N. Bartolo and M. Pietroni, Phys. Rev. D**61**, 023518 (2000); M. Gasperini, Phys. Rev. D**64**, 043510 (2001); A. Albrecht, C.P. Burges, F. Ravndal and C. Skordis, astro-ph/0107573.

[12] L.P. Chimento, A.S. Jakubi and D. Pavon, Phys. Rev. D**62**, 063508 (2000); W. Zimdahl, D. Schwarz, A. Balakin and D. Pavon Phys. Rev. D**64**, 063501 (2001); L.P. Chimento, A.S. Jakubi, D. Pavon and W. Zimdahl, Phys. Rev. D**67**,083513 (2003); A.A. Sen and S. Sen, Mod. Phys. Lett. A **16**, 1303 (2001).

[13] L Amendola and D. Tocchini-Valentini, Phys. Rev. D **64**, 043509 (2001).

[14] D. Tocchini-Valentini and L Amendola, ibid., D **65**, 063508 (2002).

[15] D. Comelli, M. Pietroni and A. Riotto, Phys. Lett. B **571**, 115 (2003).

[16] G.R. Farrar and P.J.E. Peebles, astro-ph/0307316.

[17] M.B. Hoffman, astro-ph/0307350.

[18] U. Franca and R. Rosenfeld, astro-ph/0308149.

[19] M. Gasperini, F. Piazza and G. Veneziano, Phys. Rev. D **65**, 023508 (2002)

[20] M. Pietroni, Phys.Rev. D **67**, 103523 (2003); hep-ph/0203085

[21] T. Damour, G.W. Gibbons and C. Gundlach, Phys. Rev. Lett. **64**, 123 (1990); T. Damour and C. Gundlach, Phys.Rev. D **43**, 3873 (1991); J.A. Casas, J. Garcia-Bellido and M. Quiros, Mod. Phys. Lett. A **7**, 447 (1992); R. Bean and J. Magueijo, Phys. Lett. B **517**, 177 (2001).

[22] S.M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998); astro-ph/9806099.

[23] E.I. Guendelman and A.B. Kaganovich, Phys. Rev. **D53**, 7020 (1996). Mod. Phys. Lett. **A12**, 2421 (1997); Phys. Rev. **D55**, 5970 (1997).

[24] E.I. Guendelman and A.B. Kaganovich, Phys. Rev.**D56**, 3548 (1997) ; Phys. Rev. **D57**, 7200 (1998); Mod. Phys. Lett. **A13**, 1583 (1998)

[25] E.I. Guendelman, Phys. Lett. B**412**, 42 (1997); E.I. Guendelman, gr-qc/0303048; E.I. Guendelman and E. Spallucci, hep-th/0311102.

[26] E.I. Guendelman and A.B. Kaganovich, Phys. Rev. **D60**, 065004 (1999).

[27] E.I. Guendelman, Mod. Phys. Lett. **A14**, 1043 (1999); Class. Quant. Grav. **17**, 361 (2000); gr-qc/0004011; Mod. Phys. Lett. **A14**, 1397 (1999); gr-qc/9901067; hep-th/0106085; Found. Phys. **31**, 1019 (2001).

[28] E.I. Guendelman and A.B. Kaganovich, Int. J. Mod. Phys. A**17**, 417 (2002).

[29] E.I. Guendelman and A.B. Kaganovich, Mod. Phys. Lett. A**17**, 1227 (2002)

[30] A.B. Kaganovich, Phys. Rev.**D63**, 025022 (2001).

[31] E.I. Guendelman and O. Katz, Class. Quant. Grav., **20**, 1715 (2003).

[32] R.P. Zaikov, Phys. Lett. B **211**, 281 (1988); K. Fujikawa, Phys. Lett. B **213**, 425 (1988).

[33] R. Floreanini and R. Percacci, Mod. Phys. Lett. A**5**, 47 (1990).

[34] I. Oda, Int. J. Mod. Phys. D**1**, 355 (1992); K. Akama and I. Oda, Phys. Lett. B **259**, 431 (1991); Nucl. Phys. B**397**, 727 (1993); Progr. Theor. Phys. **89**, 215 (1993).

[35] E.I. Guendelman, Class.Quant.Grav. **17**, 3673 (2000); E.I. Guendelman, Phys. Rev. D**63**, 046006 (2002); E.I. Guendelman, A.B. Kaganovich, E. Nissimov, S. Pacheva, Phys. Rev. D**66**, 046003 (2002); hep-th/0210062; hep-th/0304269.

[36] J.M. Overduin and F.I. Cooperstock, Phys. Rev. D **58**, 043506 (1998); V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000).

[37] R.D. Ball and A.M. Matheson, Phys. Rev. D **45**, 2647 (1992); L. Parker and Y. Zhang, ibid., **47**, 416 (1993); R.H. Brandenberger and A.R. Zhitnitsky, ibid., **55**, 4640 (1997).

[38] I. Dymnikova and M. Khlopov, Gravitation and Cosmology **4**, 50 (1998); Mod. Phys. Lett. A **15**, 2305 (2000). M. Nishiyama, M. Morita and M. Morikawa, astro-ph/0403571.

[39] D.G. Caldi and A. Chodos, hep-ph/9903416; B.A. Bassett, M. Kunz, D. Parkinson and C. Ungarelli, Phys. Rev. D **68**, 043504 (2003); T. Inagaki, X. Meng and T. Murata, hep-ph/0306010.

[40] N. Arkani-Hamed, L.J. Hall, Ch. Kolda and H. Murayama, Phys. Rev. Lett. bf 85, 4434 (2000); A.D. Chernin, astro-ph/0101532; Phys.Usp.44:1099-1118,2001; Grav.Cosmol.Suppl.8N2:26-32,2002.

[41] Y.Nambu and G.Jona-Lasinio, Phys. Rev. **122**, 345 (1961); W.Bardeen, C. Hill and M.Linder, Phys. Rev. **D41**, 1647 (1990); For a review see: V.A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories*, (World Scientific, Singapore, 1993).

[42] A new $6t + 6\bar{t}$ condensate has been discussed by C.D. Froggatt and H.B. Nielsen, hep-ph/0308144 and C.D. Froggatt, L. Laperashvili and H.B. Nielsen, in preparation.

[43] P.F. Gonzalez-Diaz, Phys. Lett. B**562**, 1 (2003).

[44] S. Bharadwaj and S. Kar, astro-ph/0304504.

[45] M. Milgrom, ApJ. **270**, 365 (1983); J. Bekenstein and M. Milgrom, ibid., **286**, 7 (1984).

[46] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B **511**, 265 (2001); V. Gorini, A. Kamenshchik and U. Moschella, astro-ph/0209395; M.C Bento, O. Bertolami and A.A. Sen, astro-ph/0202064; astro-ph/0210375; gr-qc/0305086; N. Bilic, G.B. Tupper and R. Viollier, Phys. Lett. B **535**, 17 (2001); astro-ph/0111325 N. Bilic, R.J. Lindebaum, G.B. Tupper and R.D. Viollier, astro-ph/0307214; J.S. Fabris, S.V. Gonzalves and P.E. de Souza, Gen. Rel. Grav. **34**,53 (2002);

H.B. Sandvik, M. Tegmark, M. Zaldarriaga and I. Waga, astro-ph/0212114.

[47] H. Bondi and T. Gold, Mon. Not. R. Astr. Soc., **108**, 252 (1948); F. Hoyle, ibid., **108**, 372; F. Hoyle, ibid., **109**, 365.

[48] R. Fardon, A.E. Nelson and N. Weiner, astro-ph/0309800; P.Q. Hung and H. Pas, hep-ph/0010126; astro-ph/0311131.

[49] R. MacKenzie, F. Wilczek and A. Zee, Phys. Rev. Lett. **D53**, 2203 (1984).

[50] W. Israel, Nuovo Cimento B**44S10**, 1 (1966).

[51] S. Weinberg, Phys. Lett. B**215**, 265 (1983).

[52] T. Damour and I.I. Kogan, Phys. Rev. D**66**, 104024 (2002); I.I. Kogan, astro-ph/0108220; V.A. Rubakov, hep-th/0104152.

[53] C.J. Isham, A. Salam and J. Strathdee, Phys. Rev. D**3**, 867 (1971).

[54] V.de Sabbata and M.Gasperini, *Introduction to Gravitation*, World Scientific (1985).